Post-Newtonian evolution of compact binary systems

Phd Thesis

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Introduction

Astrophysics based on gravitational waves is one of the new big scopes of general relativity. With the measurement of gravitational waves one has the opportunity to understand the formation of the universe. The existance of gravitational waves had already been predicted in 1916 by Einstein, who, later in 1918, also computed the so-called *quadrupole formula* which describes gravitational radiation [1]. Gravitational waves are actually "small ripples" forming in spacetime which propagate with the speed of light. Formally, they occur at the order of $\mathcal{O}(1/c^5)$, their amplitude is a dimensionless quantity the magnitude of which is of order 10^{-21} for a source of one solar mass at a distance of 300 billion light years. As general relativity is nonlinear one may raise the question whether these kind of waves really exist or not. In 1937 even Einstein disputed their existance. Eventually, the discovery of the decreasing orbital period of the B1913+16 binary pulsar by Hulse and Taylor [2] indirectly proved the existance of gravitational waves, which 'transport' energy and angular momentum from binary systems [3, 4]. Hulse and Taylor received the Nobel prize for the discovery of the B1913+16 system in 1993.

The direct detection of gravitational waves represents a great challenge for today's physicists. Detection is based on the fact that in the reference frame fixed to the detector system the relative shifts may be measured. As this reference frame is not inertial, it is sensitive to the Newtonian acceleration of the studied particles. The gravitational wave induces acceleration of the particles, proportionally to their shift. By the measurement of these shifts (or relative length variations) the polarization states of gravitational waves can be obtained, which give the independent components of the Riemann tensor. In the linearized vacuum equations there are two independent polarization states, the '+' and '$\times$' transverse states. Their name results from the fact that in the plane of the wavefront the deformational lines of force have a shape of + or 45 degree-rotated $\times$.

For the measurement of small relative length variations ($10^{-21} - 10^{-22}$) the most suitable solution is to use wave detectors which operate on the basis of the Michelson interferometer. The two largest detectors of today, which have the longest arm length, are located in the USA (LIGO, $4km$) [5] and in Italy (VIRGO, $3km$) [6]. Their frequency domain is about 10-1000 Hz with a sensitivity of about 10^{-22}. Currently, the next generation of LIGO (the Advanced LIGO) is under construction the sensitivity of which will be one order of magnitude larger than that of its ancestors.

The LISA (*Laser Interferometric Space Antenna*) [7] detector is planned to be put into orbit around the Sun after 2018, consisting of three identical spacecrafts whose positions mark the vertices of an equilateral triangle 5×10^6 km on a side. The frequency domain of LISA will be $(3\times10^{-5}-0.1)\,Hz$ with a sensitivity of 10^{-22} [8].

Nowadays, the third generational detector, the Einstein detector (ET) is being designed, which will have a similar equilateral triangle shape as the LISA detector, with side-lengths of $10km$ under the ground. According to the estimations its sensitivity may reach 10^{-24}, therefore, it may have a chance to measure gravitational waves, which are the results of the non-linearity of the Einstein equation.

Prefaces

One of the most important sources of gravitational waves are compact binaries, which consist of black holes, neutron stars or white dwarfs. These are astrophysical objects which have large "mass density", i.e., their physical dimensions are close to the Schwarzschild radius.

The equations of motion in general relativity were first given by Einstein, Infeld and Hoffmann in 1938, which meant the birth of the post-Newtonian (PN) approach [9]. This method can be applied for weak gravitational fields and slow motions. The definition of the PN parameter is $\varepsilon = Gm/rc^2 \simeq v^2/c^2$ (where m is the total mass of the two bodies, $r = |\mathbf{r}_2 - \mathbf{r}_1|$ is their distance, and $v = |\mathbf{v}_2 - \mathbf{v}_1|$ is the relative velocity) with the powers of which the PN expansion can be quantified. As an example one can mention the periastron advance, calculated for the first time by Robertson in 1938, which is a first PN order effect [10]. Nowadays the equations of motion are known up to the third PN order which can be solved by using elliptical quasi-parametrization [11, 12].

The orbital evolution of binaries can be influenced by many factors such as the rotation (hereafter spin), the quadrupole moment, and the neutron-star-like magnetic dipole moment of the bodies. The spin appears in the spin-orbit (SO) and the spin-spin (SS) interactions which are 1.5 PN and 2 PN order effects [13,14]. The leading order term for the quadrupole moment is when one constituent of the binary is a monopole moving in the quadrupole field of the other. This is called the 2 PN order QM interaction [15]. The magnetic dipole-magnetic dipole (DD) interaction appears mostly in the case of pulsars having large magnetic fields (magnetars) [16]. This contribution can be 2 PN order at most if the magnetic fields of the dipoles are at least $10^{16}G$.

Motivations and goals

My main goal was to define how the contributions resulting from the finite size of the bodies (SO, SS, QM, DD) affect the evolution of orbits, what kind of changes are caused by the linear perturbations in the signs of the gravitational waves compared to the Keplerian motion.

The equations of motion of binary systems can be decoupled into radial and azimuthal motion, therefore I discussed the two cases separately.

My research was motivated by the effects which can be derived from the spin, the quadrupole moment and the magnetic dipole moment of the bodies. It is interesting to investigate how these linear perturbations modify the motion compared to the orbits of order zero.

It is known that the rotation of the bodies is difficult to be defined according to general relativity, since the Mathisson-Papapetrou equations of motion [17, 18] pertaining to the rotating test particle are not closed. However, the equations of motion can be made closed by using appropriate gauge conditions, the so-called *spin supplementary conditions* (SSC) from which three are widely used in the literature (SSC I [19–21]; SSC II [22, 23]; SSC III [24]). My aim was to analyze spin-orbit interaction of compact binaries in these SSC gauges.

Instead of their usage for specific physical cases, the radial equations can be generalized to Kepler motions containing linear perturbations. This formalism was developed by Gergely, Perjés, and Vasúth in 2000 [25] for those cases when the linear perturbations in the radial equations are constants. This formalism can be applied for PN and SO contributions. It is an interesting question how this formalism can be generalized for the SS, QM, and DD cases where the perturbation coefficients are not constant.

As the angular motion of spin systems is considerably difficult, my goal was to examine it for the SO and PN effects.

For the classical motions of the compact binary the loss of energy and angular momentum can be computed with Einstein's quadrupole formula. It is important to determine how the finite-size contributions resulting from the SO, SS, QM, and DD can affect the parameters of gravitational radiation, namely, the frequency and the phase of the waves. It is known that by taking into account the spins, the so-called spin precession equations couple to the angular motion [26]. Therefore I aimed to determine the leading-order spin precession in the phase of the high-order (3 PN) gravitational waves.

New scientific results

I I have examined the linear perturbations of the two-body problem, namely, the purely relativistic correction of first post-Newtonian order, the SO, SS, QM and DD interactions. I described the radial equations with the help of the Lagrange formalism of these contributions, then, using linear perturbation theory, I gave the time development of the radial motion, that is, the generalization of Kepler's equation known from celestial mechanics. I was the first to determine the Lagrangian of the spin-orbit interaction in SSC II gauge, which has a substantially simpler form, than in other gauges (SSC I, SSC III). I compared the resulting dynamics with the SO results derived from the Hamilton formalism presented in the literature, and I found the two to be identical. In addition, I determined the transformation between the parametrization used in the Damour-Deruelle formalism [27] and the generalized true anomaly parametrization.

II I have studied the generally perturbed radial Kepler motions in which instead of the previously mentioned linear perturbations of constant factor I allowed for the harmonic dependence of the corrections from the true anomaly. In such radial perturbations one mostly obtains secular terms which can be given by using the generalized true and eccentric anomaly. As a result of the integration of the radial equation, $I(\omega, n) = \int \omega/r^{2+n}dt$ shaped integrals appear. Depending on the n integer number either the use of the true or the eccentric anomaly parametrization was proved to be suitable. With the introduction of this parametrization I found singular terms, for which I showed that they can be eliminated if the conditions for the coefficients of my perturbation function are fulfilled. By introducing complex variables the residue theorem could be applied and the $I(\omega, n)$ integrals could easily be computed. For a wider class of the perturbations in the radial equation I have proved that in case of $n \geq 0$ the value of the $I(\omega, n)$ integral (where ω is an arbitrary function of the true anomaly) would be equal to the residue in the origin, while in the case of $n < 0$, the value of the $I(\omega, n)$ integral would be equal to the sum of the residue in the origin and the residue in a second w_1 pole. For the previously examined SO, SS, QM and DD interactions I gave the described perturbational coefficients, and showed that the formalism I developed is fullfilled for these physically relevant cases [III].

III I investigated the effect of the leading-order PN and SO terms on the angular motion. Using the fact that the magnitude of the orbital angular momentum is constant during the dynamics, I gave the time evolution equations of the Euler angles by using the Lagrange formalism to determine the constants of motion, and thereby, I determined the time evolution of spherical polar angles in the inertial frame of the $\mathbf{J} = \mathbf{L} + \mathbf{S}$ total angular momentum. I gave the angular motion in Euler angles and determined its (secular) changes averaged for the orbital period [VI].

IV I investigated the shape of gravitational waves taking into account the contributions resulting from finite size. I derived the losses of energy and angular momentum averaged over one orbital period for a circular orbit. These were identical to the expressions for the instantaneous energy and angular momentum obtained for the Kidder type of circular orbit [14] used in the literature. In the case of a circular orbit I determined the frequency and phase of gravitational waves. I was the first to give the so-called *self-spin* interaction in the phase function, which had previously been neglected. For known binary systems (such as the Hulse-Taylor and the J0737-3039 binary) I applied the accumulated number of gravitational wave cycles ($\mathcal{N}$) until the end of the inspiralling period. Based on the Jenet-Ransom model [28] I showed that the term resulting from the self-spin is greater than the one resulting from the spin-spin interaction in a system with 'small' and 'large' spins [I].

V I investigated the high-order (3 PN) corrections to the phase of gravitational waves, i.e., the $\mathcal{N}$ the accumulated number of gravitational wave cycles until the end of the inspiralling period of a binary system modified by the equations of spin precession. Based on the equations of spin precession, I was the first to derive the equations describing the evolution of relative angles determining the κ_i and γ spin vectors in the case of QM interaction (for SO and SS interactions [29]). Based on my post-Newtonian estimations the leading order of the equations of spin precession is the precession resulting from the SO interaction. I showed that in the SS term in the phase of the gravitational wave the time-dependent correction of SO origin develops at the third PN order, which does not appear for binaries of equal mass. In the case of binaries of unequal mass this term is periodic in T_{3PNSS}, which is of order ε^{-1} greater than the period of the gravitational wave (T_{GW}). For $m_2/m_1 = 10^{-1}$ mass ratio I summed up the

SS and QM terms in the $\mathcal{N}$ number of revolution and showed that the QM term is larger than the SS term, and the SS term causes only a minor modulation. I introduced the so-called *renormalized* SS spin parameter in 2 PN order, the constant part of which causes a change in the 3 PN order on the time scale $T_{3PNSS} = \varepsilon^{-1} T_{wave}$, which is of a much more simple form, than the SS spin parameter [IV],[V].

Publications

Publications related to the theses

I **B. Mikóczi**, M. Vasúth and L. Á. Gergely, *Self-interaction spin effects in inspiralling compact binaries*, Phys. Rev. D **71**, 124043 (2005).

II Z. Keresztes, **B. Mikóczi** and L. Á. Gergely, *Kepler equation for inspiralling compact binaries*, Phys. Rev. D **72**, 104022 (2005).

III L. Á. Gergely, Z. Keresztes and **B. Mikóczi**, *An Efficient Method for the Evaluation of Secular Effects in the Perturbed Keplerian Motion*, Astrophys. J. Suppl. **167**, 286 (2006).

IV L. Á. Gergely and **B. Mikóczi,** *Renormalized second post-Newtonian spin contributions to the accumulated orbital phase for LISA sources* Phys. Rev. D **79**, 064023 (2009).

V L. Á. Gergely, P. L. Biermann, **B. Mikóczi** and Z. Keresztes, *Renormalized spin coefficients in the accumulated orbital phase for unequal mass black hole binaries,* Class. Quant. Grav. **26**, 204006 (2009).

VI Z. Keresztes, **B. Mikóczi**, L. Á. Gergely and M. Vasúth *Secular momentum transport by gravitational waves from spinning compact binaries,* Proceedings of the Eight Edoardo Amaldi Conference on Gravitational Waves (Amaldi8), J. Phys.: Conf. Ser. **228,** 012053 (2010).

Other publications

VII M. Vasúth and **B. Mikóczi** *Self interaction of spins in binary systems,* AIP Conference Proceedings **861**, 794-798 (2006).

VIII **B. Mikóczi** *Frequency evolution of the gravitational waves for compact binaries,* ASP Conference Series **349**, 301-304 (2006).

IX Z. Keresztes and **B. Mikóczi** *Kepler equation for the compact binaries under the spin-spin interaction,* ASP Conference Series **349**, 265-268 (2006).

X **B. Mikóczi** and Z. Keresztes, *Generalized eccentric vs. true anomaly parametrizations in the perturbed Keplerian motion,* Publications of the Astronomy Department of the Eötvös University (PADEU) 17, 63-69 (2006).

XI L. Á. Gergely, Z. Keresztes and **B. Mikóczi** *The second post-Newtonian order generalized Kepler equation,* Proceedings of the Eleventh Marcel Grossmann Meeting 2006, Eds. H Kleinert, RT Jantzen and R Ruffini, World Scientific, Singapore, p 2497-2499 (2008).

XII M. Vasúth, **B. Mikóczi** and L. Á Gergely, *Orbital phase in inspiralling compact binaries,* Proceedings of the Eleventh Marcel Grossmann Meeting 2006, Eds. H Kleinert, RT Jantzen and R Ruffini, World Scientific, Singapore, p. 2503-2505 (2008).

XIII **B. Mikóczi** *Elliptic waveforms for inspiralling compact binaries,* J. Phys.: Conf. Ser. **218**, 012011 (2010).

XIV M. Vasúth, **B. Mikóczi**, B. Kocsis and P. Forgács *LISA parameter estimation accuracy for compact binaries on eccentric orbits,* to be published in MG12 (2010).

References

[1] A. Einstein, Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften, **154** (1918).

[2] R. A. Hulse & J. H. Taylor, Astrophys. J. **195**, L51 (1975).

[3] P. C. Peters & J. Mathews, Phys. Rev. **131**, 435 (1963).

[4] P. C. Peters, Phys. Rev. **136**, 1224 (1964).

[5] A. Abramovici et al., Science **256**, 325 (1992).

[6] C. Bradaschia et al., Nucl. Instrum. Methods A **289**, 518 (1990).

[7] K. Danzmann, *Laser Interferometer Space Antenna: Sixth International LISA Symposium*, AIP Conf. Proc. **873** (2006).

[8] http://lisa.nasa.gov/.

[9] A. Einstein, L. Infeld & B. Hoffmann, Ann. Math. **39**, 65 (1938).

[10] H. P. Robertson, Ann. Math. **39**, 101 (1939).

[11] G. Schäfer & N. Wex, Phys. Lett. A **174**, 196; Err.-ibid. **177**, 461 (1993).

[12] R. M. Memmesheimer, A. Gopakumar & G. Sch¨a¨fer, Phys. Rev. D 70, 104011 (2004).

[13] L. Kidder, C. Will, & A. Wiseman, Phys.Rev. D **47**, 4183 (1993).

[14] L. Kidder, Phys. Rev. D **52**, 821 (1995).

[15] E. Poisson, Phys. Rev. D **57**, 5287 (1998).

[16] K. Ioka & K. Taniguchi, Astrophys. J. **537**, 327 (2000).

[17] M. Mathisson, Acta. Phys. Polon., **6**, 167 (1937).

[18] A. Papapetrou, Proc. Phys. Soc. **64**, 57 (1951).

[19] F. A. E. Pirani, Acta Phys. Polon. **15**, 389 (1956).

[20] W. M. Tulczyjew, Acta Phys. Polon. **18**, 393 (1959).

[21] W. Dixon, Nuovo Cim. **34**, 317 (1964).

[22] M. H. L. Pryce, Proc. Roy. Soc. Lond., A **195**, 62 (1948).

[23] T. D. Newton & E. P. Wigner, Rev. Mod. Phys. **21**, 400 (1949).

[24] E. Corinaldesi & A. Papapetrou, Proc. Roy. Soc. A **209**, 259 (1951).

[25] L. Á. Gergely, Z. I. Perjés & M. Vasúth, Astrophys. J. Suppl. **126**, 79 (2000).

[26] A. Apostolatos, C. Cutler, G. J. Sussman & K. S. Thorne, Phys. Rev. D**49**, 6274 (1994).

[27] T. Damour & N. Deruelle, Ann. Inst. Henri Poincaré A **43** , 107 (1985).

[28] F. A. Jenet & S. M. Ransom, Nature **428**, 919 (2004).

[29] L. Á. Gergely, Z. I. Perjés & M. Vasúth, Phys. Rev D **58**, 124001 (1998).