Randall-Sundrum type 2 brane theories and tachyon dark energy model

Phd theses

Keresztes Zoltán

Supervisor: Dr. Gergely Árpád László

associate professor

Doktoral School in Physics

University of Szeged

Department of Experimental Physics

Department of Theoretical Physics

SZTE TTIK

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Introduction

According to the newest observations the general relativity can describe the evolution of the Universe if we assume the existence of unconventional matter sources. The cold dark matter is necessary for the explanation of structure formation and the observed dynamics of galaxy clusters. The dark energy causes the acceleration expansion of the Universe. It is an important question what these dark fluids are. Which fields can explain the observations? On the other hand, a change in the gravitational dynamics can also cause similar effects to those of the dark energy and dark matter. The investigation of alternative gravitational models is also interesting from theoretical viewpoint. The string / M theory, as candidates for a unified theory, motivate more alternative gravitational models. One of these is the Randall-Sundrum type 2 brane model.

The goals of the dissertation are to investigate in detail:

- the generalized Randall-Sundrum type 2 (RS2) brane model;
- a cosmological model having dark energy as a tachyon field.

RS2 brane-world model

In RS2 brane-worlds, the gravitation acts in a 5-dimensional (5d) space-time according to Einstein-Hilbert action. In this 5d world, the observable universe is a 4-dimensional (4d) time-like hypersurface, the brane. The 5d energy-momentum tensor has a regular and a distributional part. The regular part describes the non-standard matter fields in five dimension. The distributional term containing the brane tension and the standard matter fields resides on the brane. Except gravitation, all standard model interactions and matter fields are confined to the brane.

From string theory viewpoint, our Universe is a Dirichlet 3-brane. The endpoints of open strings end on the brane, while the closed strings can move in full 5d space-time.

The brane decomposes the 5d space-time to two regions. Gluing the regions together requires the induced metric being continuous across the brane. Using the Einstein equations it can be shown that the extrinsic curvature of the brane has to jump (Lanczos-Israel equation) if distributional matter exists. Due to the brane tension the energy-momentum tensor of the brane does not vanish even in vacuum case. Therefore, the difference of the extrinsic curvatures computed both side of the brane (with the same normal) never vanishes. The Lanczos-Israel equation can also be interpreted as the jump of the extrinsic curvature creating distributional matter.

In general relativity the covariant divergence of the energy-momentum tensor disappears. The divergence of the energy-momentum tensor of standard matter generally does not vanish in

the RS2 brane-world model. However, it becomes divergence-free if the regular part of the 5d energy-momentum tensor projected onto the brane in one of its indices and into perpendicular to the brane in its other index vanishes. Otherwise, matter can accumulate on the brane from 5d regions.

The gravitational dynamics on the brane, i.e. the effective Einstein equation, can be derived from the 5d Einstein equation projected onto the brane in its both indices. New source terms arise in this equation according to general relativity. The quadratic term arising from energy-momentum tensor of standard matter can dominate at high energies in the early evolution of the Universe. The regular part of 5d energy-momentum tensor also gives new contributions. The asymmetric embedding of the brane is also a source in the effective Einstein equation. Apart from these, the non-local contributions of the gravitational effects also appear through the 5d Weyl tensor projected perpendicular to the brane in its second and fourth indices. This can give a contribution even at low energies.

Tachyonic cosmological model

Scalar fields depending only on time are perfect fluids. The barotropic index appearing in the equation of state is not a constant. One type of the Lagrange densities (L_φ) for scalar fields can be considered as the generalization of the Lagrange function of the nonrelativistic particles moving in one dimension. Another type (L_{tach}) can be derived from the relativistic particle's Lagrange function, and I will refer to this tachyon field.

Isotropic cosmological evolution can always be reproduced by minimally coupled scalar field. A similarly statement is valid for tachyon fields. Therefore, a relation can be found between the minimally coupled scalar field and the tachyon field, both leading to the same evolution of the scale factor. Thus, one can give a relation between the potentials appearing in the Lagrange densities in a sense that they could give the same evolution, provided an appropriate choice of initial conditions is made.

Background and goals

Research on generalized RS2 brane-worlds

1. The 3+1 covariant formalism using kinematical and gravito-electro-magnetic quantities was applied successfully in cosmological perturbation theory. These equations were developed for branes having symmetrical embedding into 5d space-time with cosmological vacuum. However, the generalized formalism only discusses the gravitational dynamics on the brane. Therefore, the equations do not compose a closed system from gravitational point of view. The closure can happen when the anisotropic pressure of non-local contribution of the higher dimensional gravitational effects disappears ($\widehat{\mathcal{E}}_{ab} = 0$). This makes it difficult to calculate the cosmic microwave background power spectrum. The conceptual difficulty comes from imposing boundary conditions in 5d regions.

   There also exists an s+1+1 description using canonical variables. In this formalism it was assumed that the s+1+1 dimensional space-time can be foliated by a time-like and a space-like hypersurfaces. This is not possible when the vectors in the space-time decomposition have vorticities. In general relativity the time-like vector has vorticity in Kerr or Taub-NUT exact solutions.

   Goals: (i) generalizing the above formalisms; (ii) making the brane 3+1 covariant formalism suitable for discussion of general perturbations with any brane embedding; (iii) examining whether it is possible to gain closed brane equations in a more general case than $\widehat{\mathcal{E}}_{ab} = 0$.

2. If an other vector (assigning of preferred spatial direction) exists besides a time-like vector, then it is practical to decompose the space-time into a 2+1+1 form. The local rotational symmetry (LRS) selects a spatial direction unambiguously at any point. The LRS space-time can be categorized into three classes in general relativity.

   In the RS2 model, brane solutions with local rotational symmetry were already found. These correspond to Reissner-Nordström and Kerr-Newmann space-times with a tidal charged parameter arising from Weyl tensor instead of an electric charge.

   Goal: Derivation a new brane solution having local rotational symmetry.

3. The five-dimensional Birkhoff theorem states that the 5d space-times with negative cosmological constant containing a Friedmann brane and having its symmetries along the extra dimension are static and belong to the generalized Schwarzschild - Anti-de Sitter class. This theorem is violated by Gergely-Maartens (GM) metric that has the same symmetries, but does not belong to this class. One of the coordinate transformations used in the proof of the theorem is not employable for GM metric. In hyperbolic case, it was shown that some scalars of GM solutions agree with the scalars of the degenerated horizon of extremal Schwarzschild-Anti-de Sitter having negative curvature index.

   Goal: Investigation whether the 5d Birkhoff theorem may be extensible for GM space-time.

4. The brane can radiate into 5d regions due to the interaction of particles at high energy producting gravitons which could escape into the extra dimension. The cosmological evolutions of the brane with symmetric and asymmetric embedding were considered in this setup. Another possibility is when the source of the 5d radiation is a 5d object. Standard matter is confined to the brane in the brane-worlds. Therefore, the higher dimensional radiation has to be non standard, like the Hawking radiation of 5d black holes. Hawking radiation can be imagined as particle-pair production close to the horizon. A radiation escaping from the black hole with positive energy and one going into the black hole with negative energy are created due to the high curvature. In the most generic situation, parts of Hawking radiation is absorbed and reflected by and transmitted through the brane. The motion of the brane was investigated previously when the 5d regions are Vaidya-Anti-de Sitter (VAdS$_5$) regions, where they contain black hole and the brane transmits the full radiation. However, there is only one component radiation in a VAdS$_5$ space-time.

   Goal: Investigation of the effects of 5d black hole's Hawking radiation, when the brane embedding is asymmetric and 5d regions are VAdS$_5$ regions.

5. Observations of type Ia supernovae (SNIa) implied that the expansion of the Universe is currently accelerating. The luminosity distance-redshift relation resulted by the observed light curves can also be computed theoretically in the cosmological models. RS2 brane models were confronted with SNIa dataset where relatively high value for the parameter Ω_λ were allowed. This parameter belongs to the quadratic term of the energy-momentum tensor. Meanwhile, the contribution of 5d Weyl tensor appeared as dark radiation on the brane. It was found $\Omega_\lambda = 0.026$ for the best fit. However, constraints from other cosmological and astrophysical predictions, which give smaller value for Ω_λ with many orders of magnitude, disrule this case.

   The Weyl curvature appears as dark radiation if the brane is embedded in 5d Schwarzschild - (Anti)-de Sitter space-time. An interesting case is when there is an energy transfer between the brane and the higher dimensional space-time. If the brane radiates such that the mass parameter of black holes scaling as $m \propto a^\alpha$ (where $1 \leq \alpha \leq 4$ and a is the scale

factor), the formation of structure can be explained by the Weyl fluid arising from the Weyl curvature, instead of dark matter.

Goal: Derivation of luminosity distance-redshift for RS2 brane-worlds. Comparison of RS2 model with the supernova dataset when the brane radiates.

Research on the tachyonic cosmological model

6. If we require a real potential, the tachyon fields are more special than the minimally coupled scalar fields. Then, the pressure of tachyon field is negative and the fluid can violate the strong energy condition resulting an accelerating expansion of the Universe. In the case of the potential investigated in the thesis, when its value remains real in the future, the model runs into de Sitter end state.

   However, it is not necessary to require real value for the potential. When the potential investigated in the thesis becames imaginary, other terms became also imaginary in such a way that the physical variables remaine real. The space-time does not become singular at the transition. In this domain the Lagrange density of the field corresponds to the Lagrange function of the particle moving faster than the speed of the light. The pressure of the fluid is positive, thus the expansion of the Universe is slowing down. The trajectories run into a new type of singularity known as Big Brake. The geodesic equations are not singular at the Big Brake, the geodesics can be continuated. The Big Brake is not an end state of the Universe. Due to the sudden brake, the Universe recollapses and runs into a Big Crunch singularity.

   Goal: Confronting the tachyonic cosmological model with SNIa dataset and investigating the possibility of the Universe running into a Big Brake. Investigating the evolution of this model in distant past.

Thesis points

1. *Developing new formalism for the description of Randall-Sundrum type 2 brane-world models* ([7] and [8]):

   I developed a formalism for the description of RS2 model which fits to the decomposition of the space-time into 3+1+1 form. The variables of the formalism are (i) kinematical, (ii) gravito-elektro-magnetic, (iii) matter variables. The method generalizes the brane 3+1 covariant formalism and an approximation (for $s = 3$) using canonical variables.

   I gave the generic equations, which are valid for any brane embedding both on the brane and in the outer 5d regions. I did not use any assumptions for the symmetries of 5d space-time, for embedding of the brane, and for the matter sources. The commutation relations of the derivatives and transformations of the quantities were also given for the infinitesimal change in the basis. The latter one is important in the perturbation theory.

   I derived the expression of the local 3d curvature tensor in terms of 3+1+1 variables. This is the 3+1+1 decomposition of the 3d Riemann tensor for vanishing vorticities. I derived the Friedmann equation (which is one of the basic equation in cosmology) from its corresponding contraction (from 3d Riemann curvature scalar).

   The derivation of generic 3+1+1 equations is valid in 5d regions and along the brane if the gravitational law is given by the 5d Einstein equation. They are also valid for the DGP (Dvali-Gabadadze-Porrati) model. The description was specified for the RS2 model by joining along the brane two 5d regions. These junction conditions could be generalized easily for the DGP model.

   The gravitational dynamics on the brane can be derived from a subgroup of generic 3+1+1 equations. These equations were known in the particular case of a symmetric embedding of the brane into a cosmological vacuum 5d space-time. I have corrected some mistakes in the literature. The brane equations do not give a closed system. A more generic closure condition was offered in the thesis than what was given before ($\widehat{\mathcal{E}}_{ab} = 0$) when the brane embedding is symmetrical.

   The developed 3+1+1 covariant formalism is suitable for the investigation of cosmological perturbations and for finding brane black hole solutions.

2. *Derivation of a new brane solution* ([8]):

   I have employed the 3+1+1 covariant formalism in the case of special symmetries for finding a new brane space-time. The brane is stationary and locally rotationally symmetric (LRS). The local rotational symmetry selects a spatial direction unambiguously at each points, therefore I further decomposed the brane space-time into a 2+1+1 form. The thesis specializes to type I LRS (LRS I) space-times where the time-like vector appearing in the decomposition has vorticity. From the generic equations, I have derived two coupled second order, nonlinear differential equations by imposing an anzats for the anisotropic pressure term of Weyl fluid. Due to the non-linearity of the equations, I searched for a particular solution.

   I have found a space-time similar to the charged Taub-NUT-(A)dS space-time of general relativity after a formal identification of the tidal charge with the square of electric charge. The Taub-NUT-(A)dS space-time describes an LRS I symmetric electrically charged black hole containing also a NUT (Newman-Unti-Tamburino) charge. As a consequence of the NUT charge, the space-time has closed time-like curves in some regions. The brane solution does not have electric charge, however a tidal charge appears due to the non-local effects of higher dimensional gravity. While the electric charge squared weakens the gravity attraction of a black hole, the tidal charge can make it stronger depending on its sign. The new space-time can be interpreted as tidal charged Taub-NUT-(A)dS brane.

3. *Extension of the 5d Birkhoff theorem* ([5]):

   I proved that the spherical and hyperbolic cases of the Gergely-Maartens metric describes the neighborhood of degenerated horizons of generalized Schwarzschild - (Anti)-de Sitter space-time.

   First, I considered what parameter values of Schwarzschild - (Anti)-de Sitter space-time contain two event horizons. Then, I discussed the extremal cases (when the two horizons coincide), and I derived the space-times describing the neighborhood of the degenerated horizon. These new space-times are exact solutions of the 5d Einstein equations. The horizon space-times have more symmetries than the original ones. The number of Killing vectors is the same as in the GM space-time.

   I showed that the coordinate transformations between the horizon and GM metrics exist. According to the results, the spherical and hyperbolic GM metric give the degenerated horizons of the Schwarzschild-de Sitter and Schwarzschild-Anti-de Sitter with negative curvature index, respectively. The relation is similar to what is given between the Bertotti-Robinson and the extremal Reissner-Nordström space-times.

   The hyperbolic Schwarzschild-Anti-de Sitter space-time occurs in the 5d Birkhoff theorem. The proof extends the 5d Birkhoff theorem for the GM metric in hyperbolic case, but in a weaker sense.

4. *Investigation of irradiated closed Friedmann branes* ([1] and [2]):

   I described the effects of a higher dimensional black hole radiation on a closed, asymmetrical embedded brane universe. The asymmetry was ensured by placing a black hole into only one of the 5d regions. No cosmological vacuum solution of Einstein equation is known when the space-time contains two component radiation. Therefore, it was necessary to neglect the radiation reflected by the brane. I examined the effects of the radiation transmitted across and absorbed by the brane. The 5d regions are VAdS_5 regions if the radiation is considered in geometric optics limit. I investigated the early regime of the Universe, thus the brane was radiation dominated.

   First, I considered a numerical evolution when the 5d black hole does not radiate. For curvature index $k = 1$ and brane cosmological constant $\Lambda = 0$ the radiation dominated Universe collapsed into a Big Crunch similarly as in general relativity.

   Taking the effects of Hawking radiation into account, it was found that

   - the Hawking radiation changes perturbatively the cosmological evolution with respect to the nonradiating case as two competing small effects appears: (i) the absorbed radiation increases the self-gravity of the brane leading to faster recollapse of the Universe; (ii) the pressure of the Hawking radiation pushes away the brane from the black hole, contributing to an accelerated cosmological expansion;
   - for values of transmission smaller than 0.275 critical initial brane energy densities $\widehat{\rho}_0^{crit}$ exist for which the competing two effects nearly cancel each other;
   - at larger transmissions, the critical brane energy density decreases;
   - if the initial energy density $\widehat{\rho}_0$ is smaller than $\widehat{\rho}_0^{crit}$, then the radiation pressure is dominant, while if $\widehat{\rho}_0 > \widehat{\rho}_0^{crit}$ the self-gravity caused by Hawking radiation is dominant;
   - the recollapse of the semi-transparent brane is faster for high transmissions.

5. *Derivation of the luminosity distance for flat Friedmann brane and confronting it with the dataset coming from the observations of type Ia supernovae* ([3] and [4]):

   I have derived analytically the luminosity-redshift relations in terms of elliptic integrals. The studied $\mathcal{Z}_2$ symmetrically embedded flat Friedmann branes were

   - Randall-Sundrum branes with fine-tuning (vanishing brane cosmological constant Λ). This is the original RS2 model. The luminosity-redshift relation was given by elliptic integrals;
   - the models with $\Lambda = \kappa^2\lambda/2$ (λ is the brane tension, κ^2 is the gravitational coupling constant). The luminosity-redshift relation was given in terms of elementary functions;

- the contributions arising from the Weyl fluid and from the quadratic terms of energy-momentum tensor are small and $\Lambda \neq 0$. The model has three cosmological parameters: Ω_ρ (cold dark matter), Ω_d (Weyl fluid), Ω_Λ (brane cosmological constant) satisfying $\Omega_\rho + \Omega_d + \Omega_\Lambda = 1$. For specific choice of the Weyl contribution the brane radiates. If it radiates, there is an energy transfer between the brane and the black holes of the 5d regions (LWRS model). The luminosity-redshift relation was given by elliptic integrals.

In the first two cases the 5d regions are $SAdS_5$ while in the LWRS model are $VAdS_5$. In the LWRS model the radiation escaping from the brane is absorbed by 5d black holes having increasing mass parameters as $m \propto a^\alpha$, where $1 \leq \alpha \leq 4$ and a is the scale factor.

The listed models were compared to the Gold2006 supernovae dataset. The Randall-Sundrum branes with fine-tuning were not favoured by the observations. The models with $\Lambda = \kappa^2\lambda/2$ are allowed by the supernovae dataset, however are ruled out by the low value of the brane tension, which would be in disagreement with other cosmological and astrophysical predictions.

For LWRS models with $\alpha = 0$ the best fit cosmological parameters are perfect accordance with WMAP 3-year data. The LWRS models with $\alpha = 1$ and $\alpha = 4$ agree formally with ΛCDM with the dark matter and dark energy having partially geometric origin. For $\alpha = 2$ and $\alpha = 3$ the confrontation with obeservations does not select unambigously cosmological parameters. For increasing α the allowed range of Ω_d becomes wider.

6. *Confrontation of the tachyonic cosmological model with SNIa dataset, and investigating its future and the past evolutions* ([6]):

   A cosmological model has to produce an accelerating expansion in order to fit the supernova dataset. Therefore, it was assumed that the field is in the subluminal domain at present (then it can violate the strong energy condition). The model satisfies the supernova constraints, therefore it is a new candidate for dark energy.

   I determined the parameter range where the fitting is good with SNIa data.

   If the field remains in the subluminal domain during its future evolution, the Universe reaches the de Sitter final state. By contrast some of the allowed trajectories run into the Big Brake exotic singularity in the superluminal regime. I investigated whether the SNIa dataset favours the Big Brake and found that a subgroup of the best fitting trajectories run into this new type singularity. The time scale to reach the Big Brake is similar to the age of the Universe.

   For the trajectories heading towards Big Brake the accelerating expansion has to turn into a slowing one at some part because the singularity is inside the subluminal domain. In some cases it may be possible for this transition to occur already in the recent past (for $z < 0.1$).

The best fitting trajectories with distant past indicate that the tachyon field behaved as cold dark matter. Therefore, it is conceivable that the tachyon field may provide a unified dark energy fluid model.

Publications

Publications related to the thesis

Publications in refereed journals

[8] **Z. Keresztes** and L. Á. Gergely: *Covariant gravitational dynamics in 3+1+1 dimensions, Class. Quantum Grav.* **26** 204006 (2010), e-print: arXiv:0909.0490

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[4] Gy. M. Szabó, L. Á. Gergely and **Z. Keresztes**: *The luminosity-redshift relation in brane-worlds: II. Confrontation with experimental data, PMC Phys. A* **1** 8 (2007), e-print: arXiv:astro-ph/0702610

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[A1] L. Á. Gergely and **Z. Keresztes**: Gravitational radiation reaction in compact binary systems: *Contribution of the quadrupole-monopole interaction*, *Phys. Rev. D* **67** 024020 (2003), e-print: arXiv:gr-qc/0211027

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[B11] L. Veréb, **Z. Keresztes**, P. Raffai, Zs. Udvari, M. Tápai and L. Á Gergely: *Compact binary waveform recovery from the cross-correlated data of two detectors by matched filtering with spinning templates*, (2010), To be published in the Proceedings of the Gravitational Wave Data Analysis Workshop GWDAW14, Roma, Italy 2010, Ed. F Ricci, e-print: arXiv:1005.2101

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[B7] L. Á. Gergely, **Z. Keresztes**, A. Yu. Kamenshchik and V. Gorini, U. Moschella: *Do supernovae favor tachyonic Big Brake instead de Sitter?*, to be published in the Proceedings of the Invisible Universe International Conference, Paris, 2009, Ed. J. M. Alimi, arXiv:0910.3887

[B6] L. Á. Gergely, **Z. Keresztes** and Gy. M. Szabó: *Cosmological tests of generalized RS brane-worlds with Weyl fluid*, *AIP Conference Proceedings* **957** 391 (2007), e-print: arXiv:0709.0933

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