QUANTUM INTERFERENCE IN SEMICONDUCTOR RINGS

PhD theses

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Introduction

The intensive development of computer technology and information processing did put more and more emphasis on the miniaturization of electronic devices in the past few decades. Although in the size range of present day transistors the rules of operation are still governed by the laws of classical physics, the continuing decrease of the size of the devices towards the nanometer range – where the wavelike property of the electrons can no longer be neglected – is expected to lead to the appearance of quantum mechanical effects. Due to the development of semiconductor technology, it has become possible to fabricate so-called *mesoscopic* conductors (quantum dots, quantum wires, and various other structures), which are at least in one of their dimensions comparable with the quantum mechanical wavelength of the electron. The experimentally measurable quantities of such conductors show effects, which can only be explained by using the laws of quantum mechanics. If the longitudinal size of the conductor is smaller than the phase coherence length of the electron, and its geometry is suitable to enforce the wavefunction of the electron to interfere with itself, then specific quantum effects are expected to appear. The investigation of the above-mentioned quantum transport effects, may take us closer to the production of "quantum nano-devices", the operation of which will be determined by very different rules than those of conventionally used (classical) devices, as they will be based on the utilisation of quantum mechanical effects.

The investigation of the role of the spin degree of freedom of the electron, and its possible ustilisation as a resource, has also been the subject of research related to transport in mesoscopic systems. This direction has emerged as a new field, called spin electronics or *spintronics* for short. There are numerous spintronic devices that use the electron spin as a classical resource, like the "spin valve", based on the effect of giant magnetoresistance, which is used in present-day hard drives. The more recent directions of research, related to semiconductors, however, wish to utilize the spin of the electron as a quantum resource. This idea may be related to the birth of quantum computing, which offers efficient methods for the solution of problems, that are either infeasible, or would require years to solve with classical computers. The basis of quantum information is the quantum analogue of the classical logical bit, i.e., the state of a two-level quantum system, the so-called *qubit*. As the spin of the electron is also a two-level quantum system, it can be suitable in principle to play the role of the qubit in future implemetations of quantum computing.

Scientific background

Mesoscopic devices are usually based on semiconductor heterostructures, where a narrow potential well is formed at the interface between two semiconductor layers. Here, the Fermi level lies inside the conduction band, resulting in the increase of the electron density. If the potential well is asymmetric, then an internal electric field can emerge perpendicular to the interface, which interacts with the spin of electrons moving there. The peculiarity of this so-called *Rashba-type spin-orbit coupling* is that the strength of the interaction is tunable with external gate voltage(s), which makes it very attractive for spintronic applications. An additional consequence of the potential well is that the movement of electrons perpendicular to the interface becomes quantized. For energetic reasons, in general, only one of these perpendicular modes takes part in the conduction, therefore, the electrons essentially do not move in this direction, while they are practically free to move in the sample along the heterointerface. The mean free path of electrons at very low temperatures is typically $100 - 1000$ nm. The phase coherence length, which describes the distance in which the electron, as a wave, maintains its ability to interfere with itself in spite of being scattered in the sample, is in general equal to the mean free path. (We note that the so-called spin coherence length, may be nevertheless as large as 100 μm.) The appearance of interference effects is expected when the size of the device is comparable to the phase coherence length. Such small structures can be fabricated by creating artificial potentials for the electrons by techniques such as etching, lithographic methods, or patterning by a scanning force microscope.

In the heterointerface of semiconductors it is possible to prepare *quantum rings* (that is, ring shaped quantum wires), which are smaller than the phase coherence length of electrons. These devices make it possible to investigate the effect of quantum interference phenomena on the experimentally measurable transport quantities. A frequently studied case is when the conductance of the device is measured depending on the external magnetic field. The conductance is then a periodically oscillating function of the magnetic field. This quantum interference phenomenon, besides its fundamental importance, has also been used as a basis for numerous more specific proposals, and experimental applications. It is possible, for example, to carry out phase sensitive measurements by means of this effect in a qunatum ring, where the properties of one of the arms can be changed by the presence of a quantum dot, or a local gate electrode. According to the theoretical proposals, it would be possible to use such a ring as a spin filter, or a spin switch.

If Rashba-type spin-orbit interaction is present in a quantum ring, then quantum interference depends on the strength of the spin-orbit interaction (i.e., the magnitude of the internal electric field). This is a result of the fact that the electron moving in the ring (paralel to the heterointerface), sees the perpendicular electric field as an effective magnetic field in its rest frame. The electron spin precesses around this magnetic field at an angle which is proportional to the strength of the spin-orbit interaction. As the effective magnetic field seen by the electron is perpendicular to the direction of the velocity and the electric field, its direction is different in every point of the ring, therefore, the precession angles of the electron spin in the two arms of the ring are different. By tuning the strength of the Rashba coupling with an external gate voltage, the conductance of the ring will show oscillations as a result of spin-dependent quantum interference. Such oscillations have been seen in the case of single rings, and also in the case of two-dimensional ring arrays. This quantum interference phenomenon, besides its fundamental interest, has also been used as a basis for several special (spintronic) proposals in the past few years. According to one of the proposals, a quantum ring with two external leads attached to it, in which Rashba-type spin-orbit interaction is present, is suitable to realize spin rotations that are fundamental in quantum computing, if the position of the incoming and outgoing leads, the value of the external gate voltage, and the diameter of the ring are chosen appropriately.

Motivations and goals

The ongoing intensive experimental and theoretical interest in quantum rings motivated our research regarding on the one hand, the description of the mentioned quantum interference phenomena by using a preferably simple model, and on the other hand, the investigation of their possible spintronic applications.

In the theoretical description of quantum rings it is usually assumed that the electron is transmitted from the lead into the two arms of the ring with equal probabilities. In practical applications however, the connection between the incoming and outgoing leads and the ring is not necessarily ideal (for example at the junctions the two arms of the ring are not perfectly of the same width), and the probabilites of the injection of the electron into the two arms may be different. Therefore, we wished to take into account the asymmetric injection of the electron into the two arms of the ring by using a simple model, and investigate the effect of this asymmetry on the oscillations of the conductance in the case of a ring, which encircles a magnetic flux.

One of the basic prerequisites for spintronic appliations is the preparation of spin-polarized currents. In connection to this we wished to investigate whether it is possible to polarize the spin of the electron with the help of Rashba spin-orbit interaction in a quantum ring with one input and two output terminals. For this, we aimed to solve the scattering problem of such a ring analytically, and search for an analytic relation regarding the condition of polarization between the parameters that can be changed experimentally. We also aimed to investigate the physical background of this effect.

In the above-mentioned ring —with one input and two output leads— there are two possible output terminals for the incoming electron to exit the ring, while spin-orbit interaction together with quantum interference determine the spin direction of the electron at the two outputs. We wished to study if there is quantum intertwining between the spin degree of freedom of the electron and its possible output location, i.e., its spatial degree of freedom in this ring.

A few years ago, two-dimensional rectangular quantum ring arrays were fabricated experimentally. The conductance of such arrays were measured as a function of the external magnetic field and the gate voltage (tuning the strength of the Rashba spin-orbit interaction). Motivated by the experimental feasibility, we wished to develop a simple method to calculate the conductance of such ring-arrays, in which a magnetic field and Rashba spin-orbit interaction are present. Furthermore, we aimed to study what spin directions appear at the outputs of such arrays, and in which range they are variable as a function of the external magnetic field and the strength of the spin-orbit interaction. We also wished to study how the conductance of the arrays changes if we put point-like random scattering centers into them.

Applied methods

In the theoretical description of quantum interference phenomena, quantum rings are usually considered to be one-dimensional. This approximation can be applied, when only one radial mode of the ring shaped quantum wire takes part in the conduction. This is a condition that may be fulfilled experimentally in most of the cases. In order to obtain the results presented in the dissertation, we applied two such one-dimensional models, which were suitable to determine the solution of the scattering problem of an electron in a quantum ring, and thereby, the transmission probability through the ring. Knowing the transmission probability, the conductance of the ring can be calculated by the Landauer formula, which relates the transmission probability of the device and the number of transverse modes taking part in the conduction, to the conductance. Both of the applied one-dimensional models consider electrons to be monoenergetic, thus the wave function of the electron in the leads is written as a wave of a given momentum. The reason for this approximation is the fact that experiments are usually carried out at very low (a few hundred mK) temperatures, where the energy of electrons is practically equal to the Fermi energy.

The first model assumes that the ring encircles a magnetic flux in such a way that the magnetic field is zero inside the ring. Actually, this setup corresponds to the interference experiment proposed by Aharonov and Bohm, by which the phase difference resulting form the effect of the vector potential could be demonstrated. We note however, that in actual experimental situations, the magnetic field is also present inside the ring shaped wires, contrary to the Aharonov-Bohm effect, where the field is present only inside the encircled domain, and zero along the trajectory of the electron. The model assumes the presence of elastic scattering centers in the arms of the ring and in the junctions of the leads with the ring, and describes them with the help of transfer and scattering matrices, respectively. The couplings of the incoming and outgoing leads with the ring are considered to be symmetric with respect to the two arms of the ring, but not necessarily ideal in the sense that reflections may occur at the junctions of the leads with the ring. In our own research, we modified this general model to be able to take into account the possible asymmetry between the two arms. In order to account for asymmetric injection, we determined the scattering matrix connecting the incoming and outgoing amplitudes in the junctions from the unitarity and time-reversal symmetry conditions, taking into account a proportionality constant between the elements corresponding to the two arms. We determined analytically the transmission probability by using the above mentioned matrix in the case of a ring which encircles a magnetic flux.

The other model we used in our research is somewhat more simple than the first one, as it does not assume scattering centers in the arms of the ring or at the junctions of the ring with the leads. On the other hand, this model is suitable for describing spin-dependent transport through the ring. In this model one solves the eigenvalue equation of the Hamiltonian of the electron moving in the system. The eigenvalues obtained in this way need to be equal to the energy of the incoming electron, due to the conservation of energy. The wave functions in the different sections of the ring are then the superposions of the eigenfunctions corresponding to the eigenvalue equal to the energy of the incoming electron. The model prescribes two conditions in the junctions of the incoming and outgoing leads with the ring: the continuity of the wave functions describing the state of the electron in the leads and the arms of the ring, and the conservation of the corresponding probability currents, analogously to the classical Kirchhoff's law. Thus, in this method, the occurrence of reflections at the junctions, other than those resulting from quantum interference effects, and the property of the fitting conditions, are inherently not accounted for. In the presence of Rashba-type spin-orbit interaction the problem is spin dependent: the wave functions in the leads and the different arms of the ring are two-component spinors, and one has to solve the eigenvalue equation of the Hamiltonian that contains also the Rashba spin-orbit coupling term. Here, it is important that the eigenvalue of the Hamiltonian is fourfold degenerate. The requirement of current conservation now has to be given by the spin probability currents. The resulting linear set of equations have to be solved in order to determine the transmission probability (and consequently, the conductance) of the ring. By using this model we calculated analytically the transmission probability through a quantum ring with one input and two output terminals, assuming the presence of Rashba-type spin-orbit interaction in the ring. We used this model also to determine the conductance of quantum ring arrays, where a magnetic field was present, as well. Besides the Aharonov-Bohm flux, we took into account the effect of the magnetic field also through the Zeeman-coupling, which we considered as a weak perturbation. Thereby, we were able to solve analytically the scattering problem of the individual rings constituting the arrays.

New scientific results

1. We calculated the transmission probability through two-terminal quantum rings in which the injection into the arms of the ring was asymmetric, and a magnetic flux was present in the area encircled by the ring. We showed that when no scatterer was present in the arms of the ring, then asymmetric injection led to the increase of the minima of the transmission oscillations as a function of the magnetic flux, as a consequence of incomplete destructive interference. We also showed that in the presence of a weak scatterer in the arm of the ring favored by the asymmetry, the phase of the oscillations was shifted as the phase introduced by the scatterer was changed, and asymmetry led to the shift of transmission minima to higher values. In the presence of a strong scatterer in the arm favored by the asymmetry, the phase of the oscillations was insensitive to the phase introduced by the scatterer, and asymmetry led to an overall decrease of the transmission probability [I].

2. We determined analytically the solution of the scattering problem of a ring with one input and two output leads in the presence of Rashba spin-orbit coupling that was tunable with external gate voltages. This was found to be a spintronic device capable of polarizing the spin of the electron. We showed that by appropriately choosing the position of the junctions, the value of the Rashba coupling strength, and the radius of the ring, different pure states could be formed at the outputs from an originally completely unpolarized input spin state with equal transmission probabilities [II].

3. We investigated the physical background of the spin-polarizing effect and found that polarized spin states were formed at the outputs as a result of spatial interference between those eigenstates of the ring which have the same spinor part but correspond to oppositely directed currents in the ring. At a given output this interference was destructive for a certain spin direction, while constructive for its orthogonal counterpart, which led to the appearance of the latter as a pure spin state in the given terminal [III].

4. We investigated what kind of correlations could be present between the spin degree of freedom and the spatial degree of freedom of the electron at the output of a three-terminal quantum ring. We showed that in the case when the ring polarized a completely unpolarized input, the correlations were purely classical. However, if the incoming spin state of the electron was pure, then the correlations could be purely quantum, i.e., the intertwining between these two degrees of freedom could be maximal [IV].

5. We developed a method to determine the conductance of rectangular arrays of quantum rings, in which single rings with two, three, and four terminals were used as building blocks. Using the solution of the scattering problem of these blocks we fitted the wave functions in the points where neighbouring rings touched each other, and calculated the conductance of the arrays from the resulting set of equations. We assumed that Rashba spin-orbit interaction and a perpendicular magnetic field were both present. We showed that the conductance did oscillate as a function of the wave vector, as well as a function of the strength of the spin-orbit coupling and the magnetic flux. We showed that the arrays were completely opaque for certain ranges of the wave vector, and these regions bent as the strength of the Rashba coupling increased. These non-conducting stripes were also apparent in the oscillations of the conductance as a function of the magnetic field and the spin-orbit interaction strength at a fixed value of the wave vector [V].

6. The method we developed to determine the conductance of arrays of quantum rings made it possible to investigate the spin direction of the electron at the outputs. We showed that as the electron reached the outputs of the array its spin was rotated in a nontrivial way due to consecutive spin-dependent interferences inside the device. A wide variety of spin rotations could be seen as a function of the magnetic field and the Rashba coupling strength. In order to give a more realistic description of these arrays we introduced point-like random scattering centers between the rings. We showed that certain oscillations in the conductance as a function of the magnetic field were amplified and that the conductance in the otherwise nonconducting regions increased [V].

Publications

Publications related to the theses

[I] P. Vasilopoulos, **O. Kálmán**, F. M. Peeters, and M. G. Benedict: *Aharonov-Bohm oscillations in a mesoscopic ring with asymmetric arm-dependent injection*, Phys. Rev. B **75**, 035304 (2007).

[II] P. Földi, **O. Kálmán**, M. G. Benedict, and F. M. Peeters: *Quantum rings as electron spin beam splitters*, Phys. Rev. B **73**, 155325 (2006).

[III] **O. Kálmán**, P. Földi, M. G. Benedict, and F. M. Peeters: *Spatial interference induced spin polarization in a three-terminal quantum ring*, Physica E **40**, 567 (2008).

[IV] **O. Kálmán**, P. Földi, and M. G. Benedict: *Quantum and classical correlations of spatial and spin degrees of freedom in quantum rings*, Open Syst. Inf. Dyn. **13**, 455 (2006).

[V] **O. Kálmán**, P. Földi, M. G. Benedict, and F. M. Peeters: *Magnetoconductance of rectangular arrays of quantum rings*, Phys. Rev. B **78**, 125306 (2008).

Other publications

[VI] **O. Kálmán**, and M. G. Benedict: *Quantum gates and protocols with spherical Wigner functions*, Int. J. Quantum Inf. **3**, 501 (2005).

[VII] P. Földi, **O. Kálmán**, M. G. Benedict, and F. M. Peeters: *Networks of quantum nanorings: Programmable spintronic devices*, Nano Lett. **8**, 2556 (2008).

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[X] P. Földi, M. G. Benedict, **O. Kálmán**, and F. M. Peeters: *Quantum rings with time-dependent spin-orbit coupling: Spintronic Rabi oscillations and conductance properties*, Phys. Rev. B **80**, 165303 (2009).