On Heat Death in Past, Present or Future

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A lot of paradoxes have been invented about the time evolution of the entropy of the Universe. A family of them is called Heat Death, and on the basis of these paradoxes it was often doubted if the entropy of the Universe can at all defined, or if it can, would it obey the Second Law, &c. Indeed, Universe is an exceptional system (solitary, no neighbourhood, no conservation of energy) and therefore the concept of the entropy of the Universe needs some caution. Also the history of this entropy is complicated. The present paper discusses these problems.

A hőhalálról múltban, jelenben vagy jövőben. Paradoxonok sokaságát állították már fel az Univerzum entrópiájának idővolúciójáról. Ezeknek egy családját hőhalálnak nevezik és ezen paradoxonok alapján gyakran kételkedtek abban, hogy lehet-e egyáltalán az Univerzum entrópiáját definiálni, vagy ha lehet, engedelmeskedni fog-e a Második Törvénynek, stb. Valóban, az Univerzum egy kivételes rendszer (társtalan, nincs szomszédsága, nics energia-megmaradása), és ezért az Univerzum entrópiájának fogalma bizonyos körültekintést igényel. A története is bonyolult ennek az entrópiának. A jelen tanulmány ezeket a problémákat vitatja meg.

1. Introduction

Paradoxes constitute a *negative* of collecting knowledge. At a given stage of the science have a given amount of knowledge; some correct, some not. Then we try to find out the result of a Gedankenexperiment. A paradox is found if two different results can be obtained by seemingly correct and consequent derivation, or if the only result is impossible or contrary to common sense. Paradoxes are signals that *something* is wrong: either our present theories or data (which is always possible) or our common sense (which is the essence of our previous experiences). Paradoxes are not to be taken too seriously; they do not indicate onthological problems. However they are to be kept in mind, because until they are not solved, something must be wrong.

A famous old paradox of cosmology was Heat Death. In the present decades it is rather forgotten, and indeed it has been *partially* solved. But not fully, and it is worthwhile to remember it from time to time.

Heat Death was originally formulated in the last century. After establishing the first two Fundamental Laws of thermodynamics it seemed that they contradicted each other when applied on the whole Universe. For a closed system the First Law states the conservation of energy, while the second the equilibration or by other words the monotonous increase of entropy. The conservation indicates infinitely long past and future of the Universe. But if so, then the entropy should have reached (or asymptitically approached) its final value and been utterly equilibrated (rotten or corrupted) which is not so. Here was the paradox.

Many attempts were made to solve it. But, as again and again turned out, the thermodynamics of the *whole* universe needs extra caution. Namely, the Universe (if such a definite entity exists at all) palys the role of the proverbial sick horse of veterinarian courses. It may be infinite; it is thermodynamically not closed even when it is a closed hypersphere; the energy is not conserved in it; and so on. In this paper we show up several problems and try to give the solutions if available. We shall see that we do not know to give a complete description of the thermodynamic evolution of the Universe from a very exotic (singular?) past to the possibly infinite future. However our present knowledge about the ultimate past and future is hazy enough to be able to *imagine* ways out of the paradox.

2. The pre-Friedmannian era

The classical explanations were various, diverse and not statisfactory. The simplest one stated that is ill-founded to extend the thermodynamics of finite systems to the infinite Universe. Either because the latter is not a *closed* system, or because it is not *macro*scopic but *mega*scopic.

However consider a Universe homogeneous on large scale (to be conform with the cosmological principle). Then there is no net currrent between any two sufficiently large neighbouring parts and then these parts are as if they were physically closed. Remember this for later use. So infinity in itself does not help.

The appeal to megascopy was more philosophical. It is a popular idea to classify objects onto three levels in sizes, masses, particle numbers or anything else, with disjoint laws on each level. The first such level would be microscopy, i.e. the „atomic" level, the second macroscopy, which is our own level, and the third megascopy starting somewhere at very large astronomical systems and „ending" with the global Universe. For example Poincaré guessed [1] that the random motion of *planets* is not analogous to that of *molecules*; while the second is undoubtedly heat, the first is rather mechanical work because of the larger sizes and distances.

Such a trichotomy is by no means impossible, since our present laws are approximate. Indeed, such a separation of levels was the popular solution of the measurement paradox of quantum mechanics, although now it seems that it is not hopeless to derive the different behaviours from common laws [2], [3], [4], [5]. For a recent review about possible levels see Ref. 6. However never anybody observed astronomical processes making the entropy decrease. And if one fundamental law of thermodynamics ceases to exist above macroscopic level, why just the Second Law, why not the First, for example?

Eddington suggested a solution completely possible but very disturbing. In the past of an infinitely old system *any* kind of thermodynamic fluctuations must have happened, the larger the rarer. Life and intellect become possible sufficiently far from equilibrium, so we can exist only not too long after a sufficiently large fluctuation; therefore the extreme rarity of such an event is not incompatible with the fact that we observe its consequences. When a very large fluctuation happens, afterwards somebody comes into existence and observes that Heat Death is still far ahead; when the system have returned to near-equilibrium and the paradox does not appear, nobody exists to be contented. This was the strongest classical solution. However with the advent of General Relativity the whole context changed.

3. The Friedmannian past singularity

Einstein introduced the idea that the (geometrical) laws of the space-time are governed by the matter in it. Ignoring all the details, there are 3 and only 3 geometries for the space-time which are homogeneous and isotropic in space. They are the **Robertson—Walker** Universes [7], namely

$$\begin{aligned} ds^2 &= dt^2 - R^2(t)\{dx^2 + F^2(x)\,d\Omega^2\} \\ d\Omega^2 &\equiv d\Theta + \sin^2\Theta d\Phi^2 \\ F(x) &= \begin{cases} \sin x & \text{if } k = +1 \\ x & \text{if } k = 0 \\ \operatorname{sh} x & \text{if } k = -1 \end{cases} \end{aligned} \tag{3.1}$$

Henceforth the time units are such that the light velocity c is unity. The sign constant k is called space curvature. The $k = +1$ Universe is closed, the 3-surface of a 4-hypersphere of radius R (and then the full volume is $2\pi^2R^3$); the other two geometries are open spaces and $R(t)$ is a scale length of the geometry. Therefore the global geometry is determined by the sign constant, while the local curvature can change according to R.

From the Einstein (or **Einstein—Hilbert**) equation of gravity, for this geometry filled with a homogeneous fluid matter one gets two evolution equations. They can be written as

$$\dot{R}^2 = -k + (8\pi/3)GR^2\epsilon \tag{3.2}$$

$$\dot{\epsilon} + 3(R/R)(\epsilon + P) = 0 \tag{3.3}$$

where ϵ is the energy density and P is the dynamical pressure.

This system of equations is not closed. A further relation would be needed among R, ϵ and P, whose simplest form is an equation of state $P = P(\epsilon)$ say in local equilibrium. Then the equations can be integrated and there remain only a few constants of integration in the solution. For demonstration we give here the solution in the simplest case: $k = 0$ and $P = 0$ *(dust)*. Then

$$R = W(t - t_o)^{2/3} \tag{3.4}$$

$$\epsilon = (1/6\pi G)(t - t_o)^{-2} \tag{3.5}$$

Here two constants of integration appear, but W can be made 1 by rescaling the coordinate x as seen from eq. (3.1) for $k = 0$. The other constant is simply the startpoint of time counting and it can be made 0 by a convention.

Now observe that this solution has a singular moment when all the distances are 0 and the density is infinite. One cannot uniquely continue the solution through a singularity. First **Friedmann** gave the dust solutions for all the three k's: all contain somewhere a singularity and for $k = +1$ singularities are periodic in time. So the history either starts or ends with a singularity or happens from singularity to singularity. With pressure the history may be complicated, but the **Hawking-Penrose** singularity theorems [8] predict past or future singularities for a very wide class of equations of state.

The galactic redshifts indicate $\dot{R} > 0$ for the present, this means the existence of a *past* singularity; the future one is doubtful, since it exists only for $k = +1$. Then the Universe is not infinitely old; the estimated age from redshifts is cca. $15 \cdot 10^9$ years, in rough accord with observed chemical compositions &c. Therefore it seems that the paradox does not exist anymore.

This is not quite sure. The so called *horizon problem* [9] states the fact that from the Beginning in a standard cosmology no physical signal could have traversed betwen some remote parts of the Universe now observable for us, and in spite of this they have e.g. the same temperature. Sometimes an infinite quantum prehistory is suggested at **Planck** density $\epsilon \sim (c^7/\hbar G^2)$ where the gravitational law is unknown enough for the existence of a static solution [10]. But then returns the infinite age and past Heat Death. And the future fate remains anyways a Heat Death, except for the finite future of the $k = +1$ closed Universe, which, in turn, ends with a catastrophe. So, independently of the past, it is worthwhile to discuss the infinite future of the $k = 0$ and -1 Universes. According to scientific folklore they asymptotically go to a Heat Death. The remaining part of this paper is devoted to the question if this asymptotic Heat Dying is the necessary fate or not.

4. On the Pfaffian form of thermodynamics

Some years ago my attention was called to the fact that even without the past singularity and finite age the Heat Death paradox would not be necessary [11], from at least two different thermodyynamic reasons. The first one is very simple. For simplicity, consider a closed $k=+1$ Universe. It is not a closed system *in thermodynamic sense*. Namely, a thermodynamic system is closed if *all* its independent extensive quantities (volume, energy, particle numbers &c.) have constant total values in it. However, in our Universe R is increasing. So for the *whole* Universe V is not constant. If a thermodynamic system is not closed, then the Second Law does not guarantee the entropy growing, and, furthermore, it does not guarantee that the possible maximum of the entropy be a finite constant. In our case the total entropy may grow in the growing volume, but not necessarily will asymptotically approach a final value. We will return to this point later.

The second thermodynamic complication is even more interesting, and it would have operated even in the infinite Euclidean Universe of the last century. Therefore now let us concentrate on this point. It is interesting to contemplate why it did not appear as a solution in the earlier literature. My guess is that this was caused by a historical accident. Namely, the remark states the fact that the existence of irreversibility in thermodynamic processes does *not* a priori guarantees the growth of entropy for more than two independent extensives, and volume, energy and particle number are already at least three. If the **Caratheodory** construction had been made *before* the General Relativity, then this solution should have appeared in the discussions.

Thermodynamics is the physics of irreversibility, say, the irreversibility of energy transfer. Consider a thermodynamic system with the independent extensives X^i, $i \leq n$. In the familiar systems these extensives are the volume V, *internal* energy E, various particle numbers N^α, &c. There seems to be no problem with the definition of V or N; the internal energy E needs some considerations because it is not necessarily the total energy, but now let us first assume that it is already defined.

Let us move infinitesimally in the state space $\{X^i\}$ by adding some amounts of the extensives to the system. Then there is a dE; some part of it is reversible, some not. Let us write then the decomposition

$$dE = \delta W + \delta Q \tag{4.1}$$

the first is the *mechanical work*, the second is the *heat transfer*. Generally nonr of them is an infinitesimal of a function Q or W, this is the reason for the notation δ.

Now we are interested only in the *irreversible* part. In arbitrary processes of course δQ can arbitrarily change. However, define a *special* kind of isolation called *adiabatic* (say, no „heat" transfer) and require that such processes have some irreversibility. The irreversible nature of infinitesimal *adiabatic* processes can be formulated by

$$\partial Q \geq 0 \qquad (4.2)$$

(„non-compensated heat"). For a given system of course the structure of ∂Q is fixed:

$$\partial Q = Z_r(X^i)dX^r \qquad (4.3)$$

where and throughout the whole paper the Einstein convention is followed, i.e. there is summation for indices occurring twice, above and below. Then with the actual form of the functions Z_i (characteristic for the actual system) (4.3) can be reduced to one of the canonical **Pfaffian** forms [12]:

$$\begin{array}{ll} \partial Q = dQ(X^i) & K = 1 \\ \partial Q = T(X^i dS(X^i) & K = 2 \\ \partial Q = dU(X^i) + T(X^i)dS(X^i) & K = 3 \\ \partial Q = H(X^i)dU(X^i) + T(X^i)dS(X^i) & K = 4 \end{array}$$

and so on, $K \leq n$. *The* $K = 2$ case is the usual thermodynamics *A* tedious but straightforward derivation [12] results in the following statements:

For $K \leq 2$ Cond. (4.3) leads to *global* irreversibility, namely generally between two points of the state space (except on a hypersurface of 0 measure) adiabatic processes can go only in one direction and not back. However it is not quite so for $K \leq 3$. There between *any two* points one can go on one path forward and on another backward. So now the irreversibility of elementary steps $\partial Q > 0$ does not result in global irreversibility and perpetua mobilia of second type are possible.

The complete proof is in Ref. 12. However a rough argumentation goes as follows. For $K \leq 2$ (if the functions T and S have unique values and do not change signs) $\partial Q > 0$ means increase of S, so different points in the state space. But for $K = 3$ $\partial Q > 0$ does not necessarily indicate anything monotonous in S and U.

Now a number of experiences point against the existence of perpetua mobilia. Postulating their nonexistence $K \leq 2$ follows and the existence of nontrivial temperature rules out $K = 1$. Therefore $K = 2$, and then we arrive at **Caratheodory**'s thermodynamics [13].

However this, and therefore the increase of the entropy, is based on the nonexistence of perpetua mobilia. If this leads to a paradox and the final result is unacceptable for somebody, he may assume that $K \geq 3$ (if the number of independent extensives is ≥ 3), only $\partial Q \approx TdS$ in a good approximation under macroscopic usual circumstances.

Since the expanding Universe the matter is more and more diluted and no exotic behaviour is predicted, we do not want to propose this as a solution of the paradox *in the future*. However, a $K \geq 3$ Pfaffian form may have been valid in the quantum past of the Universe, of which we know nothing at all [14].

Henceforth we *assume* that $K = 2$.

5. Is the growing thermodynamic entropy unique?

So let us assume that the entropy S cannot decrease in a closed system (or in adiabatic processes). Then, as we saw in the discussion of the Pfaffian forms, S is a function of the independent extensives, therefore

$$dS = (\partial S/\partial X^i)dX^i = Y_i(X)dX^i \tag{5.1}$$

where Y_i are the entropic intensives. In axiomatic thermodynamics these intensives possess equal values in the equilibrium of two different systems. Since Y_i are the partial derivatives of S, therefore if we know the form of the single function $S(X^i)$, we know the full thermodynamic behaviour of the system [15].

However one cannot a priori know the S functions. By pure thermodynamic measurements one may try to determine the S_α functions observing extensive values X_α and X_β at equilibria $Y_\alpha = Y_\beta$. For ν different systems this sequence of measurements imposes $1/2\nu(\nu-1)n$ relations on the νn partial derivatives, so at sufficiently high ν one expects even overdetermination, and if not, then unique entropy functions. However it is not so. For $\nu > 3$ no further independent relations are obtained [16], and thermodynamic measurements cannot completely determine the function $S(X^i)$.

The maximum of determining it is the freedom [17]

$$S_\alpha (X^i) \rightarrow K^2 S_\alpha(X^i) + C_r X^r \tag{5.2}$$

where K and C_i are *universal* constants. Indeed, such a change of gauge rescales and shifts the corresponding intensives of *all* systems in a synchronised way and the equilibria remain equilibria. So the thermodynamic entropy is not unique.

Henceforth for simplicity we restrict ourselves to systems with 3 independent extensives, V, E and N. Then the freedom is

$$S_\alpha \rightarrow K^2 S_\alpha + AV + BE + CN \tag{5.3}$$

Then it is easy to see that even statistical physics does not help. E.g. **Boltzmann**'s H theorem singles out such $H = -S/V$'s, whose changes are monotonous. Now in a closed system V, E and N are fixed, so they do not influence monotonous changes. Similarly, e^S is proportional to statistical weights, but the transformation (5.3) do not alter the **Riemann** geometry of the state space [17], so e.g. fluctuation probabilities are invariant [18].

In a closed system (5.3) is irrelevant. However in a Universe where V changes because of expansion and E because of energy nonconservation, the change of S depends on the choice of the actual S. With an „appropriate" A it is even possible that one S increases and another decreases.

In Ref. 19 we suggested that the universal constants A, B &c. should be determined by observing the Universe. This way is possible, because the *thermodynamic* pressure p

$$p = (\partial S/\partial V)/(\partial S/\partial E) \tag{5.4}$$

depends on A and B. By assuming that the thermodynamic pressure p is the leading term of the dynamical pressure P of the matter filling the Universe, and taking some probable Ansatz for the matter (say, ideal gas), A and B influence the scale function of the Universe $R(t)$, so from the observed expansion A and B can in principle be deduced. At present the best available constraints are:

$$\begin{aligned} &-10^{-6}\,cm^{-3} < A < +10^{+3}\,cm^{-3} \\ &0 < B < 10\,MeV^{-1} \end{aligned} \tag{5.5}$$

where $A = 0$, $B = 0$ belong to the „naïve" gauge (say, $p = nT$ for ideal gas, &c. [19].

So if we are fanatic to know what is the fate of the entropy of the Universe, we can determine first which is the its most proper entropy. But it is always told that the Caratheodory construction leads to unique entropy and temperature. What is then this variety of entropies?

6. On the decompositions of heat and work

Observe that thus far we have postulated only the *existence* of irreversibilities in thermodynamics. Since we are interested in the ultimate fate of the whole Universe here, some efforts must not be spared and one ought to take as little in face value as possible. So: we do not see perpetua mobilia of second type, although they have been looked for very hard. So thermodynamics seems to have *global* irreversibilities. However, in a system any wanted state can be prepared by adding or subtracting just the proper amount of energy, particles &c. to or from a system. So it is rather hard to formulate ireversibility or an *open* system; and of course it is impossible to do for a *closed* one, since extensives do not change at all in such ones. Then we require an inequality

$$\partial Q \geq 0 \tag{6.1}$$

for changes in *adiabatic* systems as told in Sect. 4. Adiabatic isolation means roughly „no heat transfer" or „only mechanical work transferred" [12], but at this point neither heat nor thermal energy have yet been defined, so it would be better to find a physical limiting procedure towards adiabatic isolation.

Now, let us accept a well motivated such limiting procedure. Then we can perform experiments to check the predictions of the formalism. We have a container as adiabatically isolated as possible or needed. The internal energies of states can then be mapped via a **Joule**-type experiment. Take a state $V = V_o$, $E = E_o$ (this is an arbitrary definition for an initial point), $N = N_o$. Adiabatic walls permit only mechanical work to be transferred, and no „heat" can leave the system. Then let some mechanical work be injected into the fluid in the container. After some time internal turbulences vanish. The total volume and particle number remained unchanged, so only E can change. Therefore the only possibility is $\partial E = \partial W$, and ∂W was measured outside. So we define the internal energy of the new state as $E_o + dE$, and if this in equilibrium with another piece of the same matter then E/V is the same that too.

Now we know V, E and N in any specific state, and we *know* that $\partial Q = T(E,V,N)dS(E,V,N)$ must hold, otherwise perpetua mobilia would exist (the Pfaffian form $K = 1$ is a special case here with $T \equiv 1$). This means a *foliation* in the space $\{E, V, N\}$, i.e. that the function S has a unique value at any point [12], and then for adiabatically isolated systems we are at the Second Law. Adiabatically isolated systems continuously climb up from layer to layer in S.

This S must be the entropy mentioned in the thermodynamic axioms [15] because that is the quantity with the tendency of growing in closed or adiabatic systems. That we know how to measure (see Sect. 5). There remain some freedoms in it, but they do not influence the foliation since the additive terms cannot be multivalued. So from the Pfaffian form all of them may be entropies; if other principles rule out some of them, the fewer the better.

Now we can start with the checking process. If indeed we see the foliation in S, everything is settled. But what if we see something else? Then something fundamental must be wrong. It cannot be in the Pfaffian form since no perpetuum mobile is seen. It cannot be in the determination of the internal energy, because that process is unique. There remains a single possibility. Either the 3 variables $\{E, V, N\}$ are insufficient for description, which is theoretically possible, but only a technical problem, or the physical limiting process of adiabatic isolation was improper. Since there *is* global irreversibility, there must be at least one construction for adiabaticity which at the end will result in good foliation.

Let us stop briefly at this point. It seems as if we had a full-proof construction. Take a container with strong, thick and rigid walls, with a negligible hole and turbine inside. A rope through the hole connects the turbine with a weight outside. By the rope we can put mechanical work into the container and if the walls are more and more substantial, nothing is expected to leave the system, so no heat either, anything be heat. This is the naïve idea about a limiting Joule experiment, and it sems unique.

However, this limiting procedure is not necessarily unique, because

ad 1) one cannot guarantee that *nothing* leaves the container even with infinitely thick isolating walls, remember e.g. gravity which cannot be shielded;

ad 2) infinitely thick walls of a finite container can absorb finite amount of, say, energy, without even the possibility of detecting the absorption, i.e. if nothing comes out, still something may leave the sysytem under investigation.

Therefore there is no guarancy that the definition of *adiabatic* isolation compatible with global irreversibility would be unique. If not, different possible definitions result in different, but equally possible thermodynamic descriptions of systems. And the same is true for the freedom (5.2).

However, even then infinitely many description are ruled out. This means that indeed the existence of global irreversibilities prescribe how to decompense the energy transfer dE into heat transfer $\eth Q$ and mechanical work $\eth W$. And remember that for $K = 2$ Pfaffians there is no function $Q(E, V, N)$ which would yield dQ as differential. For $K = 1$ it would exist and would coincide with the entropy; then the temperature would always be the same constant. According to everday experiences of the last centuries, this is not the situation.

In cosmology the large scale homogeneity guarantees that no net current of *anything* physical can go between two great expanding parts of the matter. Therefore we can be sure about the lack of current of internal energy even before defining the internal energy. Then there may be a physical limiting process for adiabatic isolations, because no walls are involved. If so, then the Universe can define its natural thermodynamics.

7. Can entropy really approach its final value in a closed system?

In this Chapter we take a finite amount of the matter of the Universe, put absolutely isolating walls around (which do not necessarily exist, but that comes in the next Chapter), and ask, what will happen. The folklore tells that the entropy of that piece of matter will asymptotically go to its maximal value. There is no proof *against* this belief, but an example will demonstrate that it is not necessarily so.

Imagine a container of volume of some liters, made from indestructible material, with a realistic internal partition (say a thin sheet-iron). Put some indestructible detectors inside, and say, one mole of oxigen to the left and two moles of hydrogen to the right, if you like, with some amount of catalysers. Since the container is indestructible, it is the same if you bury it or not. The detectors are continuously detecting, what will be seen by the sequence of generations?

Our knowledge is finite in this moment. According to this finite knowledge the story goes as follows.

Step 1: Possible initial temperatures equilibrate by heat conduction, even through the partition. Time: seconds, increase in specific entropy: say *01*.

Step 2: At the same temperature there is a pressure difference between the two sides of the partition. The difference is slowly bending the sheet-iron. After, say, weeks the partition is bent in such an extent that volume changes equilibrate the pressures. The specific entropy has gone up by ~ 1.

Step 3: After that chemical potentials are still different on the two sides, since on the left there is no hydrogen, on the right no oxygen. But in a time between a year and a millenium the sheet-iron gets rusty and perforated (evidence: archeology). Through the holes the gases diffuse and completely fill the whole container. Specific entropy goes up by ~ 1. Now all the intensives have become spatially homogeneous, so one would expect that the entropy is already maximal. However it is not.

Step 4: H und O can be combined into H_2O, and they are being, with a rate depending on temperature and catalysers. So as time goes by, there will be *three* kinds of molecules, H_2, O_2 and H_2O, each with its own chemical potentials, and, say, in hundred thousend years they equilibrate as

$$\mu_{water} = \mu_{ox} + 2\mu_{hyd} \tag{7.1}$$

Again entropy went up by ~ 1. *The story ended here for a physicist in 1890.*

Step 5: Nuclear fusion still can go. Without catalysers cold fusion takes a time much longer than billion years and according to the present stage of knowledge no efficient catalyser of cold fusion is known (the palladium has not been proven sufficient). However the container is everlasting. So at the end the matter is a mixture of electrons and various nuclei, mostly Fe^{56}, with the equilibria

$$\begin{aligned} \mu_{He} &= 4\mu_H \\ \mu_{Fe} &= 56\mu_H \\ &\text{\&c.} \end{aligned} \tag{7.2}$$

With the differences of rest energy deliberated the temperature goes up to ~billion *K*; the specific entropy again went up by *~1. Here ended the story for anybody in 1960.*

Step 6: Grand Unification predicts proton decay e.g. according to the scheme $p \rightarrow e^+ + \pi$. Experiments are going to check it. If so, in a time $\sim 10^{32}$ years most nuclei vanish, and the remainders equilibrate according to eq. (7.2) and to a new equation

$$\mu_H + \mu_{e+} \tag{7.3}$$

Again the specifiv entropy went up by *~1, and the story ends here for us at the present.*

But a new step has been conjectured since 1960. Therefore nothing rules out new steps to be discovered. The scheme was roughly the same increase of specific entropy at the appearance of each new degree of freedom. In an infinite time will there be finite or infinite such steps, will the final entropy be finite or infinite?

Ignoramus et ignorabimus. In finite time we can discover the possibility of finite steps, so the *actual* answer is always finite. But this does not prove anything. One could argue that after proton decay there is not too much remaining rest energy to thermalize. However *note that this rest energy was unknown in 1890, therefore everybody would have used the same argument against further entropy production beyond Step 4.* We do know energies beyond rest energy, say the zero pont fluctuation of quantum electrodynamics. The present theories do not suggest anything for ways of deliberating this infinite energy, but this is not necessarily the final stage of knowledge.

Therefore we cannot definitely decide if there is a (finite) maximum of the entropy of a closed container or not. If not, the entropy may grow forever, without reaching any maximum even asymptotically. Of course, it is possible that the entropy increase is *practically* nil for aeons.

8. Can closed containers physically exist even as limits?

Here we very briefly show an example when the favourite closed systems of thermodynamics are physically impossible under some circumstances. Our example is the quantum field effect of the expansion. Only the results are mentioned; for the details see Ref. 20 and citations therein.

Quantum field theories in their present forms are not necessarily compatible with General Relativity. However they can be used in a curved time-dependent geometry, e.g. in the metric (3.1), and then one gets a time-dependent nonzero energy density even for the vacuum. It is very roughly similar to a thermal radiation with

$$T \sim \hbar \dot{R}/R \tag{8.1}$$

One may visualize the result in the way that in a changing geometry everything is excited at least by the specific energy (8.1).

Then this energy will appear even in a closed container. One may tell that such a container is not closed. However the effect causing the energy to appear is the change of the metric, and this is called colloquially gravity. Gravity cannot be shielded because of the equivalence principle. Therefore if geometry is changing, here are no containers of fixed volume in which the energy of the system could be kept constant.

This is a situation in which we *know* thet no complete isolation is possible, which was conjectured in Chapter 6. Still the thermodynamic formalism can work. One may, of course, have some doubts about a formalism based on physically impossible abstractions. But this a question deserving lengthy and elaborate discussions. For the present we may remain at the usual thermodynamics.

9. The lack of conservation laws

A *closed system* is a system in which E, V and N are constant. This is a thermodynamic definition, but there are some beliefs that E and N should be conserved in a properly circumwalled system. Let us first concentrate on E.

The previous Section showed an example when no physically possible walls can keep the energy constant. But Sect. 6 mentioned that in the Universe it is possible to single out a container without walls which is absent of currents crossing the borders. Such a „container" is a part of the Universe, whose fictitious borders were drawn at some t_o and afterwards follow the expansion. So, due to complete spatial symmetry no net current flows through the borders. Now V grows, so the system is not *closed* in thermodynamic sense, but at least there is no *transfer* of E. However, if there is no energy *current*, still there may be energy *source*. And indeed, there is. Consider eq. (3.3). Thence

$$(\epsilon V)^{\cdot} = -P\dot{V}; \; V \equiv V_o(R/R_o)^3 \qquad (9.1)$$

Therefore the volume integral of ϵ, which cannot be anything else than the internal energy E, is definitely not conserved.

Eq. (9.1) is *not* some new result of General Relativity; it is the usual equation of hydrodynamics, or can be written as $dE + PdV = 0$, which is the usual formulation for *quasistatic* adiabatic processes. However, outside the scope cosmology matter configurations have boundaries. For a volume extending beyond the boundaries in the source equation $P = 0$ (vacuum) and then the total energy is conserved. But in Universe P is homogeneous in space, therefore one cannot take volumes with conserved energy. The only exception is if the pressure is everywhere 0, i.e. all the matter is without any interaction. The present Universe is fairly close to such state, but this statement is not general at all.

As for N, various particle numbers exist, some do not seem conserved, some do. However, from time to time it turns out that a particle number is only *approximately* conserved. E.g. in the $SU(3) \cdot SU(2) \cdot U(1)$ Standard Theory of particle physics there are 3 conserved charges: baryonic number B, leptonic number L and electric charge Z. (In fact, B and L are conserved for three disjoint families.) In the $SU(5)$ simplest Grand Unification, however, B and L are not conserved separately, only $B - L$. The corresponding predicted effect is the proton decay, for which experiments are still running; with no clear result, but the predicted lifetime is very long under present circumstances. So we cannot a priori know which particle number is conserved and which is not.

In addition, according to any knowledge, for large comoving volumes of the Universe $Z = 0$, and it is not impossible that $B - L = 0$ as well. If so, then the conserved numbers take trivial values in the Universe.

10. The final enumeration of doubts

Let us try to describe then the thermodynamic evolution of a large expanding volume of the Universe. There are no currents. The isolation seems adiabatic. Global irreversibilities then must appear. However there is not a single closed system in the whole description. V is growing, E is changing and maybe a lot of N's as well. Then what will happen with S in such an autonomous volume?

The answer is by no means trivial. From pure thermodynamic viewpoint it is not too important either: what is important that is the global irreversibility. It may seem paradoxical again that S might decrease with irreversibility, but the system (or the whole Universe, either) *is not closed in thermodynamic sense*.

The simplest demonstration can go through the transformation (5.3). Consider a model Universe of perfect fluid; no viscosity, no nonequilibrium processes, &c. Then one expects $S = 0$. But if so, after the transformation S can grow or decrease according to the actual values of the constants A and B.

Still in such cases one can keep $S \geq 0$ if the particular entropy is chosen in accordance to the thermodynamic leading part of the hydrodynamic pressure (remember Sect. 5). Indeed, this entropy, and therefore the corresponding specific values of the free constants in the tranformation (5.2), are not selected by Thermodynamics, but by the Universe. It would deserve some further study if such an entropy can always be selected even for general kinds of matter.

Note that with an increasing energy growing entropy of course does not necessarily implies asymptotic equilibration. Consider some temperature inhomogeneities in the system. Some energy flows from the hotter place to the cooler one, so there is energy transfer inside the system. However if energy is being produced, then it is by no means impossible that in spite of the transfer the difference is maintained by the new energy. Since in some cases the energy production is proportional to the entropy production [21], some pattern-forming processes analogous to those in the thermodynamics of open systems may have appeared in the early vehement stages of evolution.

11. The possible fates of Universe

First about Beginning. For the Universe, being unique, it would be better not to have the freedom of initial conditions. There is a hope for such a unique initial condition. In the unified theory of Gravity, Relativity and Quantization Planck data formed exclusively from G, c and $\hbar$ would be unique and would describe natural „elementary" objects or states. Such a unified theory is not at reach, although supergravity or superstring theory may be fair attempts to get at it.

Even incomplete unifications may show up „natural initial conditions". It was mentioned in Sect. 3 that static solutions might exist and can be got from „semiclassical approximations" at **Planck** density $10^{93}\ g/cm^3$ and Planck radius

$$R_{Pl} \sim (\hbar G/c^3)^{1/2} \sim 10^{-33}\ \mathrm{cm} \qquad (11.1)$$

[10]. Another example is to incorporate **Hawking** radiation into the thermodynamic description and energy-momentum tensor [20]. In some models then the past of the

Universe becomes geodesically incomplete, and suddenly appears with a radius in the order of R_{Pl}. Until we get the complete theory, we can *hope* in natural initial conditions.

However in that stage within the volume $\sim R^3$ the total entropy was ~ 1. Now $S/N \sim 10^8$ (deduced from data of the $3\ K$ black-body radiation), and R seems to be $\gtrsim 10^{28}$ cm. Therefore $S \sim 10^{87}$ [9]. So, if the natural initial conditions were true, very strong entropy production must have happened in some early stages. In accordance with this, originally E in a volume $\sim R^3$ was $\sim E^{Pl} \sim 10^{16}$ erg, and for radiation-dominated Universes eq. (9.1) would imply *decrease* of E, while now it seems to be 10^{53} erg. E can have increased only with *negative* P. Luckily, some irreversibilities make P decrease. A variety of models with substantial negative pressure and roughly exponential expansion was invented (see e.g. Refs. 9 and 22); this stage may have happened at $t \sim 10^{-35}\ s$, $10^{28}\ K$ temperature. Afterwards the Universe may have undergone a lot of phase transitions, irreversibilities &c. [23], but for bulk properties the evolution was not too different from that of a simple radiation-filled Universe.

Radiation dominance ended at a time when the radius was $\sim 10^{-3}$ of the present one. Afterwards gravity could form density inhomogeneities, evolving through steps of fragmentation. The final step is contraction into stars; they start to produce energy in nuclear fusion, and then contraction stops. For stars above several solar masses this equilibrium is only temporary, after say 10^9 ys the nuclear fuel is exhausted and the star collapses; some part shrinks into singularity and the periphery is ejected. However less massive stars can remain in equilibrium as white dwarfs or neutron stars „forever".

Therefore more and more matter is put away in compact cold objects, therefore stars become more and more dispersed. This more or less resembles the Heat Death of the last century. However the *ultimate* fate depends on the sign constant k in eq. (3.1).

For $k = +1$ the Universe is spatially closed. Such models with simple pressure laws always result in a recontraction. Present observations are not yet enough to decide the value of k, but even if $k = +1$ we cannot be close to the recontraction. Rough estimations indicate at least 50 billion years future; towards the end of this period practically all the stars will be minute red dwarfs under half solar mass. At the end the recontraction will dissolve all the structures. All guess-works of calculation suggest that at the end S will be in the same order of magnitude as now (except if in the new quantum density era a $K > 2$ Pfaffian form develops and then emerging perpetua mobilia decrease back S to its natural value ~ 1, ready for a new cycle.

For $k = 0$ or $k = -1$ all known models predict expansion forever. Forever is a serious word and our finite present knowledge is insufficient to calculate forward for infinite time. However we can predict for finite times: the longer the less safe. Again, stars are dying. After some say 10^{12} ys there are practically no shining stars, no temperature gradients. However still there are cold compact objects, so there are serious gradients in chemical potentials. They cease to equilibrate because cold neutron stars and blackened dwarfes are gravitationally bound objects. During that period S remains practically the same as now.

However, *if Grand Unification is correct*, the number of the present protons will be halved at 10^{32} ys from now by proton decay. That is almost total conversion of the proton mass, so S will go up (according to a very rough estimate) by a factor ~ 1000. This means reappearance of temperature gradients as well at the decaying stars. (These gradients will be much more moderate than the present ones: energy deliberation rates of stars will then be 10^{-20} times the present ones.) The increase of temperature gradients will not contradict to the Second Law: in the same time the chemical potential gradients decrease because of the disappearance of baryons. This will be an excellent example of cross effects of equilibration.

Afterwards there will be no stars anymore; the resulting positrons will annihilate with the electrons. Bound configurations of massive neutrinos may remain if neutrinos have mass at all. There is one more predicted energy producing process on longer time scales and that is the Hawking radiation of black holes; for the smallest black holes created in astrophysical collapses this time scale is very roughly 10^{60} ys, but the existence of such a radiation still would need confirmation from the unified theory. Even if it existed, it would leave S in the same order of magnitude.

Beyond that time present theories do not tell anything; remember Sect. 7.

12. Conclusions

This paper is bold enough to discuss ultimate questions even if only about entropy. Therefore there is great chance fot mistakes. However we can draw the conclusions that i) there is no a priori reason to be sure about the possibility of a Caratheodory-type entropy construction for the Universe in its whole life, but ii) if good arguments (as e.g. well-founded lack of perpetua mobilia of second type) exist for it then the entropy of the Universe can be defined; iii) this entropy will then be thermodynamically quite regular, but its evolution may be influenced by the fact that the Universe is not a closed system in thermodynamic sense. In addition, the Universe may have infinite future and iv) one cannot guarantee the finite final value of the entropy for infinite times even in a closed system.

As for the future of Universe, a story can be and has been told, but one cannot correctly predict for infinite times.

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